The President’s Commission on
The United States Postal Service

Analysis of the Postal Service’s
Logistics Network and Development
of a Network Optimization Model

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Summary

AT&T undertook a task to develop an analysis model for the logistics network of the United States Postal Service (USPS) to support the President’s Commission on the United States Postal Service. The Commission desired to know the excess capacity of the processing and distribution system, how it could be optimized, and what the financial impact of the optimization would be. AT&T’s approach was to first develop a simple model that disregarded mail shapes and classes. The simple model would reflect the as-is system, allocating costs to transportation and processing. The simple model would be used to perform a linear-programming optimization. Second, AT&T would develop a more advanced model that captured the processing capacity at mail-processing plants by class and shape of mail and that identified the mail flowing between plants so alternative networks could be evaluated.

An analysis model was developed that would model the plant operations and transportation. However, adequate data to populate the model and validate it was not available in sufficient time to perform the analysis. Meetings with USPS representatives indicate that some of the required data do not exist at USPS Headquarters and the data that are available need to be validated. This is due to the fact that at least part of the logistics network has evolved locally with managers at all levels making agreements and negotiating to achieve acceptable results. These efforts are not visible to Headquarters, and therefore are not able to be modeled without a data collection effort at the plants.

In order to make strategic decisions about network improvement, the USPS will need to collect the kinds of data described in this report. The USPS may have already done this as part of the USPS Transformation Plan. USPS is sponsoring the Network Integration Alignment (NIA) project that is modeling the USPS logistics network. Since AT&T was to perform an independent study, AT&T did not have access to validated and processed data from the NIA project nor did AT&T utilize specific insights from the NIA project.
Introduction

Executive Order 13278 established the President’s Commission (Commission) on the United States Postal Service for the purpose of examining the state of the United States Postal Service (USPS), and to prepare and submit a report articulating a proposed vision for the future of the USPS and recommending the legislative and administrative reforms needed to ensure the viability of the USPS. The Commission is to consider: 1) the role of the USPS in the 21st century and beyond; 2) the flexibility that the USPS should have to change prices, control costs, and adjust service in response to financial, competitive, or market pressures; 3) the rigidities in cost or service that limit the efficiency of the postal system; 4) the ability of the USPS, over the long term to maintain universal mail delivery at affordable rates and cover its unfounded liabilities with minimum exposure to the American taxpayers; 5) the extent to which postal monopoly restrictions continue to advance the public interest under evolving market conditions, and the extent to which the Postal Service competes with private sector services; and 6) the most appropriate governance and oversight structure for the USPS.¹

The Commission requires specialized assistance in analyzing the Postal Service’s current logistics network, including its processing, distribution, and retail operations. This project was to develop a network model that could be optimized while maintaining the existing universal service obligations and determine the economic benefits to the USPS if its distribution network were modified to reflect the optimized network from the model.

AT&T set out to build an as-is model of the distribution of mail among the 380 centers and facilities and the associated processing load for each. A separate cost model would be built to reflect the fixed cost of each center and the variable costs associated with processing the mail and distributing the mail. Distribution costs would include the distribution of mail to the local post offices and carriers. After determining the existing workload, the model would be adapted to show the capacity of each center and facility to process additional mail. AT&T was to then examine alternatives that consolidated workload in existing centers and facilities in order to reduce the number of centers and facilities. Finally, AT&T was to determine the impact on costs, both fixed and variable, that these changes would cause, realizing that transportation costs could decrease due to better utilization of vehicles or could increase due to longer travel distances caused by bypassing the closed centers/facilities.

The USPS has undertaken its own study of the logistics network, the Network Integration Alignment (NIA) project, as part of the USPS Transformation Plan. The AT&T analysis was to avoid reuse of NIA processed data and results.

Methods, Assumptions, and Procedures

AT&T was to build a network model with the nodes representing centers and facilities and links representing transportation routes. Additionally, the network would have an additional node at each center/facility to represent the local post offices as an aggregate. Separate from the network model, AT&T was to develop an economic model that represented the cost of processing and distributing mail for each center/facility. The cost model was to include processing costs and transportation costs. Processing costs would apply at the nodes and could be divided into variable and fixed costs. The variable cost changes with the volume and, to some extent, with the composition of the mail. Fixed cost is the remaining cost that would not change with mail volume or composition. Transportation costs would be calculated along the links between nodes and would represent the cost of distributing the mail.

Initially, the model was to represent the existing USPS logistics network. The model was to be tested by determining the shortest delivery times between nodes in the model to validate that the results were consistent with existing service performance. Then the model would be modified to determine whether the work load at some nodes could be combined at a single node while still meeting service requirements.

Even though the revenue would not be calculated for increased or decreased demand, the model would be stressed and tested for sensitivity to cyclical demand and plant degradation to ensure that the optimized system could deliver according to the existing service standards.

Network Model

Nodes in the network represent processing centers and facilities. The nodes model the inputs, outputs, and processing at the center/facility. As input, each center/facility can receive mail from three sources: collections, delivery from other centers/facilities, and drop ship mail from business customers. The mail is processed and sorted to different levels, then is distributed from the center/facility. Output from the center/facility can be sent to local post offices and carriers for delivery, or to another center/facility.

The links between nodes represent transportation, both cost and time. There can be more than one link between a pair of nodes where each link represents a different mode of transportation.

Appendix 1 describes a simple linear programming model for optimization. The linear-programming model could be run to determine an optimum number of tons of mail to allocate to each route connecting centers/facilities and selecting which centers/facilities to use for processing. It should be noted that this approach depends on some simplifying assumptions. These include:

a. Capacity of a processing and distribution centers can be obtained in weight

b. Cost (processing & shipping) per ton for movement of mail can be obtained for each activity
c. Mail requirements for each 3-digit community can be obtained.

d. No distinction is to be made for the various classes of mail as costs are against weight of processing and movement. The emphasis is on the efficiency of the distribution system.

e. All costs are attributed to the movement of mail into, through, and out of each plant to the next processing center or post office.

The USPS logistics system is sufficiently complicated that a linear programming model would not capture all of the constraints and individual idiosyncrasies required to find a solution that could be implemented. Even though the simple model would disregard service standards in assumption d, it is expected that service standards for classes of mail form a significant impact upon the postal system. However, by applying the simple linear programming model to the postal service logistics network, an order of magnitude solution could be obtained that would inform the Commission about the expected improvements.

After the linear programming solution, a second model would be developed that takes into account the different classes and shapes of mail. Centers/facilities could be parameterized based upon their capacities for different shapes of mail and individual constraining factors. The basic capability of a center/facility is based upon the processing machinery available. Because each center has its own idiosyncrasies, each must be examined individually to determine its capacity to process mail and parameterized appropriately.

Two key model components are needed to represent each plant: the incoming mail load and the plant’s internal processing capacity. The incoming mail model separates the load into individual mail streams. The streams are based upon shape and class of service. Only major mail streams are modeled. There are several low volume classes of service that are not considered in the model, for example the relatively small stream of registered mail. Neither does the model consider residual flows such as postage due, missent, or undeliverable mail.

The model assumes that all mail incoming to a plant, neglecting residual flows, leaves the plant in the same day. It is believed that this is the case for a typical day. However, it is realized that under extremely heavy incoming variations, this may not be the case.

Incoming mail is grouped into three categories: collection, pre-processed, and bulk. It is divided into streams based upon shape and class of service. The shapes modeled are:

- Letters and cards
- Flats
- Parcels

The classes of service modeled are:

- First Class Mail
• Priority Mail
• Express Mail
• Standard Mail
• Package Service

Periodicals is an additional, important class of mail subject to rigorous service standards, but are not included in the model. The impact of Periodicals Class of Service should be considered.

Not all classes are available for all shapes. The eleven streams resulting from shape/class combinations are as shown:

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Priority</th>
<th>Express</th>
<th>Standard</th>
<th>Package</th>
</tr>
</thead>
<tbody>
<tr>
<td>Letters</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Flats</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Parcels</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Incoming mail is also modeled to have varying degrees of pre-processing. Collected mail can be commingled. Some separation of streams is accomplished at collection; basically pre-culling into separate shapes and classes. Large mailers, third party mailers and other processing plants provide large quantity of presorted mail. The model considers four levels of presort:

• Carrier Route/Firm
• DDU/DSCF/ADC/AADC
• 5 digit zip
• 3 digit zip

The incoming mail is modeled as a time distribution over a twenty-four hour processing day.

**Economic Model**

The economic model for this project primarily consists of a cost model. Because the mail volume to be processed is fixed, the model will not vary the revenue. The cost model consists of costs for transportation and costs for processing. In the simplest model, it can be assumed that the cost for processing will remain the same regardless of where the processing takes place. That is, as long as there is excess capacity at a center, then the cost of processing an additional letter at that center/facility is assumed to be identical to any other center/facility. While this is not precisely true, it is a useful approximation to obtain an order of magnitude estimate of cost changes. The changes in this simple model would be entirely due to transportation choices.

Each link in the model represents one mode of transportation between centers/facilities. The mode used would depend upon the service standards for that portion of the mail. For
example, two-day mail would most likely travel by air while standard mail could possibly travel by rail or truck.

After the simple model was developed, a more complex model would have been developed that would take into account the different costs for processing at each center/facility.

**Data Requirements**

In order to develop a Logistics Network Optimization Model for the USPS, a significant amount of data and understanding of the current postal delivery system is necessary. The following type of data is needed by Distribution Center:

1. Cost, revenue, and volume data by 3-digit zip code
2. Distribution Center Capacity data such as:
   a. Number and volume of trucks presenting in-bound mail to each center
   b. Tons of mail in-bound to the center
   c. Center capacity of mail sorting equipment
   d. Size of population serviced by each center
   e. Volume of mail per unit of population serviced by each center
   f. Number of employees assigned to each center
3. Inter-distribution center mail flow information:
   a. Distance in miles between centers
   b. Ton-miles of mail arriving from other centers
   c. Cost per ton-mile for moving mail to each zip code serviced
4. Detailed variable cost data by center
   a. Postmasters
   b. Supervisors & Technicians
   c. Clerks & Mail handlers
   d. City Delivery Carriers
   e. Etc.
5. Fixed cost data by center
   a. Facility costs
   b. Utilities overhead
   c. Depreciation of truck fleet
   d. Depreciation of other capital equipment
Results and Discussion

The process used for obtaining data for this project consisted of questions submitted to the office of Corporate Audit and Response Management at USPS. The office would either answer the question directly or locate the expert at the USPS Headquarters who could answer the question. Answers were reviewed by the USPS Law Department to determine the level of confidentiality and required protection of the data. Initially, all data were delivered in hard copy. Subsequent to the first delivery, data were delivered in electronic format. Certain data used for the NIA project were restricted from access due to concerns about patent infringement and the need to obtain an independent assessment. However, raw data collected for the NIA project were provided. Most data were Postal Service specific and additional meetings were scheduled to meet with the data originators, excepting the NIA project, to explain and describe the data. Problems with data collection precluded implementation of the analysis models within the period required.

Network Model Results

The existing state of the USPS logistics network has evolved locally. Each center/facility, cluster, and area manager has made decisions to accommodate the required processing. Facilities themselves differ as they have been adapted to differing requirements. This means that capabilities (and sometimes processes) can vary significantly by center/facility.

The processing centers/facilities sort mail to the finest level possible, with the goal to sort to the carrier delivery route sequence. Each carrier would receive his mail in the order it would be delivered on his route. Because the limiting factor in sorting mail is the number of sorting bins on a machine, each center/facility has a different capacity for the number of routes for which it can sort. As a result, one center could sort mail down to carrier level for mail going to another center/facility, to enable it to bypass the first pass at the receiving center/facility. On the other hand, if the excess capacity did not exist, then the mail might only be sorted to the three-digit or five-digit zip code level. This means that the actual processing is split across more than one center/facility, increasing the complexity of the model (and the data collection).

The data available at the headquarters’ level of the USPS has been collected to meet USPS requirements. In the case of cost data, the information is used for establishing postal rates. Therefore, data is primarily collected based upon class of mail and is aggregated as it passes up the management hierarchy. Costs are collected by categories and not identified by center/facility.

In the case of mail input to centers and transported between centers, the data is collected for purposes other than analysis. For example, arriving mail is measured by varying means. Trucks arriving at the plant are individually recorded by size and percentage full. However, the interpretation of percentage full varies by the collector. Sometimes, trucks with palettes that occupy all of the floor space will be called 100 percent full, even if they are not stacked to the roof. At other times the estimate of the percentage would take into
account unused overhead space. Some of the data appeared to include a volume estimate for the mail rather than the truck. These factors caused the volume input data to vary widely and is the main reason that density calculations to convert from volume to pieces of mail could not be performed consistently or reliably.

There were often ambiguities in the data, based upon the understanding of the person asked to collect the data. As a result, the data is not consistent from center to center. Additionally, data for delivery between centers/facilities is not kept by piece or weight. It is known how many trucks arrive at each center and how many trucks depart, but in general, there is no measure of the amount of mail that transits between particular centers/facilities.

Mail processing and sorting is measured by the piece. Several attempts were made to correlate arrivals, by volume, with processing, by piece. Densities seemed to be unreasonable and were not consistent across different samples. For example, the data supplied suggested that for the Northern Virginia Processing and Distribution Center there were 10 letters per cubic foot. This would be an extraordinarily low count representing extreme circumstances, not representative of a “typical day” and was thus assumed to be erroneous.

Because the required data was either not available or flawed, we were unable to calibrate and validate our network model. While it would be possible to collect and validate the necessary data given adequate time and resources, it is beyond the scope and resources of this task.

### Economic Analysis Results

<table>
<thead>
<tr>
<th>Mail Class</th>
<th>Revenue ($000)</th>
<th>Total Vol-Variable Contribution To Fixed Costs</th>
<th>Pieces</th>
<th>Weight (lbs)</th>
<th>Contributopn To Fixed Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>First class</td>
<td>$36,479,064</td>
<td>$18,833,364</td>
<td>102,378,632</td>
<td>4,283,649</td>
<td>$0.1840 $4.3966</td>
</tr>
<tr>
<td>Priority</td>
<td>$4,720,100</td>
<td>$1,433,200</td>
<td>998,151</td>
<td>1,875,147</td>
<td>$1.4359 $0.7643</td>
</tr>
<tr>
<td>Express</td>
<td>$910,467</td>
<td>$453,987</td>
<td>61,280</td>
<td>59,086</td>
<td>$7.3983 $7.6730</td>
</tr>
<tr>
<td>Other -- Mailgrams</td>
<td>$1,356</td>
<td>$584</td>
<td>2,757</td>
<td>2,757</td>
<td>$0.2118 -</td>
</tr>
<tr>
<td>Periodical</td>
<td>$2,164,863</td>
<td>($69,337)</td>
<td>9,689,758</td>
<td>4,006,072</td>
<td>($0.0072) ($0.0173)</td>
</tr>
<tr>
<td>Standard</td>
<td>$15,818,700</td>
<td>$5,728,500</td>
<td>109,230,636</td>
<td>10,315,522</td>
<td>($0.0067) ($0.0553)</td>
</tr>
<tr>
<td>Packages</td>
<td>$2,080,070</td>
<td>$1,853,800</td>
<td>1,075,086</td>
<td>3,690,639</td>
<td>$0.2105 $0.0613</td>
</tr>
<tr>
<td>Special</td>
<td>$2,629,300</td>
<td>$1,122,800</td>
<td>424,929</td>
<td>87,502</td>
<td>$2.6423 -</td>
</tr>
<tr>
<td><strong>Subtotal Domestic Mail</strong></td>
<td><strong>$64,803,920</strong></td>
<td><strong>$27,728,748</strong></td>
<td>201,861,229</td>
<td>24,317,617</td>
<td><strong>$0.1374</strong> <strong>$1.1403</strong></td>
</tr>
<tr>
<td>Fixed Costs</td>
<td>($29,009,300)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Domestic Mail</strong></td>
<td><strong>$64,803,920</strong></td>
<td><strong>$66,084,472</strong></td>
<td>($1,280,552)</td>
<td>201,861,229</td>
<td><strong>$0.0063</strong> ($0.0527)</td>
</tr>
</tbody>
</table>

SOURCE: USPS Cost & Revenue Analysis for FY2002

#### Exhibit 1: Current Revenues & Costs by Postal Class

While individual costs for centers/facilities could not be determined, there are some trends that can be discerned. A top-level economic analysis of the Postal Service revenues and costs is conducted by the USPS on an annual basis. The results of that analysis for FY2002 are summarized in Exhibit 1: Current Revenues & Costs by Postal Class. As Exhibit 1 indicates, Postal Service revenue exceeds allocated variable costs

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* Postage rates were raised by an average 7.7% on June 30, 2002 and were in effect for only three months of the fiscal year. If these rates had been in effect for the entire fiscal year it would have made a significant difference in the contribution to fixed costs, especially for classes of mail with small contributions in FY02.
for all domestic classes of mail except Periodicals. However, when fixed costs for facilities, capital equipment (e.g., mail sorters, delivery trucks), and carrier street time are taken into account the Postal Service lost nearly $1 billion in FY2002. From reviewing the *USPS Cost and Revenue Analysis for FY2002*, it is apparent that the variable costs are covered by revenue pricing while the fixed costs that are not easily traceable to a class of mail remain unallocated to any class of mail. These unallocated costs represent 43% of the total costs of the Postal Service.

From Exhibit 1, one can see that first class and standard mail make up over 93% of the piece volume of the mail service and nearly 60% of the weight of all mail delivered. The contribution of these two classes of mail cover over $24.5 billion of the fixed costs of postal operations. Hence if their volume were to decline significantly, there would be a reduction in the coverage of fixed costs from these key revenue generators and a potential increase in Postal Service losses.

Exhibit 2 was taken from the USPS Chief Financial Officer’s presentation to the President’s Commission and it shows four years of continuous decline in the volume of single-piece First Class Mail. Single-piece First Class Mail accounts for about half of total First Class Mail. From 1998 through the year 2002, this mail volume declined by about 10%. Should this trend continue, the amount of fixed costs covered by this revenue generator would decline. In order to remain profitable either postal rates would need to be increased or mail delivery costs would need to be reduced.

When the Postmaster General looked at standard mail volume, he found another picture of decline beginning in 2001 as the US economy began its slide into a period of recession and high unemployment. For two consecutive years, 2001 and 2002, there has been a decline in Standard Mail. Again, with a decline in Standard Mail volume, there is a reduction in the fixed costs covered by the revenue generator.
A key component of single-piece First Class Mail is bill remittances. As reported by the National Automated Clearing House Association (NACHA), a trade group whose mission includes promoting online bill paying, “during the first quarter of 2003 consumers paid more than $48 billion worth of bills online using the association’s clearing house, the ACH Network.” “Online bill payment is now a mainstream consumer activity,” said Elliott C. McEntee, president and CEO of NACHA. The association estimates that online bill payments for 2003 will exceed $200 billion, compared to $96 billion in 2002, according to Michael Herd, spokesperson for Virginia-based NACHA. While these 500 million transactions represents less than .5% of the current First Class Mail volume, e-business transactions are a growing industry and will ultimately cause a reduction in the volume of First Class Mail processed by the USPS.

<table>
<thead>
<tr>
<th>Mail Class</th>
<th>FY2002 Pieces</th>
<th>Contribution/Piece</th>
<th>10% Reduction By 2010</th>
<th>10% Reduction Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>First class</td>
<td>102,378,632</td>
<td>$0.1840</td>
<td>10,237,863</td>
<td>$1,883,336</td>
</tr>
<tr>
<td>Priority</td>
<td>998,151</td>
<td>$1.4359</td>
<td>99,815</td>
<td>$143,320</td>
</tr>
<tr>
<td>Express</td>
<td>61,280</td>
<td>$7.3983</td>
<td>6,128</td>
<td>$45,337</td>
</tr>
<tr>
<td>Other -- Mailgrams</td>
<td>2,757</td>
<td>$0.2118</td>
<td>276</td>
<td>$58</td>
</tr>
<tr>
<td>Periodical</td>
<td>9,689,758</td>
<td>($0.0072)</td>
<td>968,976</td>
<td>-6,934</td>
</tr>
<tr>
<td>Standard</td>
<td>87,230,636</td>
<td>$0.0656</td>
<td>8,723,064</td>
<td>$572,850</td>
</tr>
<tr>
<td>Packages</td>
<td>1,075,086</td>
<td>$0.2106</td>
<td>107,509</td>
<td>$22,627</td>
</tr>
<tr>
<td>Special</td>
<td>424,929</td>
<td>$2.6423</td>
<td>42,493</td>
<td>$112,280</td>
</tr>
<tr>
<td><strong>Subtotal Domestic Mail</strong></td>
<td>201,861,229</td>
<td>$0.1374</td>
<td>20,186,123</td>
<td>$2,772,875</td>
</tr>
</tbody>
</table>

**Exhibit 3: Impact of Potential Further Reductions in Mail Volume (000’s)**

Many observers project further declines in both first class and standard mail due to the rise of e-mail communication, e-business contracting activity, on-line bill paying, and alternate means of advertising. As the technology revolution continues over the next 10 years, one can postulate the reduction in coverage of the current fixed costs of the Postal Service. Using the information in Exhibit 1 for cost per piece of mail delivered and making an assumption on the percentage reduction in volume, Exhibit 3 illustrates the potential adverse impact on the economics of Postal Service operations. Note, while Exhibit 3 hypothesizes a 10% reduction in the volume of all classes of mail, it is used here only to illustrate the potential reduction in coverage of fixed costs associated with further reductions in mail volume. Exhibit 3 shows that a 10% reduction in mail volume could potentially result in further reducing the coverage of fixed costs at the current postage rates. That reduction would be over $2.7 billion additional debt incurred annually by the Postal Service unless a more efficient network of distribution and delivery can be developed. The $2.7 billion represents about 4% of the annual operating (fixed and variable) costs of the Postal Service measured in FY 2002 dollars.
Recommendations

Because the USPS logistics network has evolved locally, it has many local optimizations that are not readily visible to outside observers. USPS will need to collect the kinds of data identified in this report in order to consider global optimization or improvements, if it has not already done so as part of its NIA efforts. It is unclear whether USPS has already collected this data for the NIA project, as AT&T was not allowed access or insight into the NIA project. Clearly the economic data indicate that if First Class and Standard Mail continues to decrease, the deficit between cost and revenue will eventually grow and continue to grow over time. Improvements to the logistic network is one area that should be considered to lower costs.
Appendix A

Example Linear Programming Transportation Optimization

The simplest form of efficiency model that appears to apply readily to the Postal Service is the classic linear-programming “transportation model.” The model has numerous economic and business applications that have nothing to do with the original transportation problem solved by F. L. Hitchcock in 1941. (See *Journal of Mathematics and Physics*, **20**: 224-230 (1941)). The essence of the Postal Service efficiency problem may be conveyed by a simple example adapted from *Linear Programming and Economic Analysis*, by Dorfman, Samuelson, and Solow. Suppose the Postal Service has three distribution centers. We shall call them A, B, and C. Further, suppose these distribution centers supply mail to five localities, which we shall call 1, 2, 3, 4, and 5. Also, suppose the cost of processing and shipping a ton of mail from each distribution center to each locality is known and fixed. Using the data for this example given in Exhibit 4, the problem is to find a pattern of shipments that involves the least possible total transportation costs for the system given the constraints in Exhibit 4. Clearly a decision is needed to determine the number of tons (if any) to be shipped by each distribution center to each locality. Note, this decision could be based on daily volume, weekly volume, monthly volume, or annual volume and may be different for different levels of demand on the system based on seasonal patterns or population density. For simplicity, we assumed an average daily volume for the example in Exhibit 4.

### Postal Service Capacity & Requirements in Tons of Mail

<table>
<thead>
<tr>
<th>Locality</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10</td>
<td>$20</td>
<td>$30</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>$15</td>
<td>$40</td>
<td>$35</td>
<td>115</td>
</tr>
<tr>
<td>3</td>
<td>$20</td>
<td>$15</td>
<td>$40</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>$20</td>
<td>$30</td>
<td>$55</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>$40</td>
<td>$30</td>
<td>$25</td>
<td>70</td>
</tr>
</tbody>
</table>

**Capacity (Tons)**: 50, 100, 150, 300

Exhibit 4: Postal Service Transportation Problem

According to Exhibit 4, the capacities of the three distribution centers are 50, 100, and 150 tons of mail per day. The quantities to be shipped to each locality are given in the last column. The transportation costs per ton are given in the body of the exhibit. Thus the cost of shipping a ton of mail from Distribution Center “C” to locality “1” is $30 per ton. It is important to note that the total capacity of the three distribution centers just equals the total requirements for mail from all addressee consumers.
This problem may now be set up in linear-programming form subject to three constraints. The constraints are:

1) Shipments planned for each distribution center must not exceed the capacity of that center.

2) Shipments to each locality must equal the requirements of that locality

3) Shipment values cannot be negative.

In mathematical form, the problem is to minimize total cost, $T$, subject to these constraints. That is:

Equation 1: \[ T = \sum_i \sum_j c_{ij} x_{ij} \quad i = A, B, C; \quad j = 1, 2, 3, 4, 5 \]

Where $x_{ij}$ is the non-negative number of tons shipped from distribution center $i$ to locality $j$ and $c_{ij}$ is the cost per ton for shipment between the two so that $T$ represent the total cost of operating the system. This is to be minimized subject the constraints given in equations 2, 3, and 4.

Equation 2: \[ \sum_j x_{ij} = k_i \quad i = A, B, C; \quad k_i = \text{capacity of center } i \]

The first constraint implies the total shipments planned for each distribution center must not exceed its capacity. The second constraint implies that shipments to each locality must equal the requirements of that locality. Hence we have:

Equation 3: \[ \sum_j x_{ij} = r_j \quad j = 1, 2, 3, 4, 5; \quad r_j = \text{requirements for locality } j \]

Finally, the third restriction is that the value of all shipments must be non-negative. That is:

Equation 4: \[ x_{ij} \geq 0 \quad i = A, B, C; \quad j = 1, 2, 3, 4, 5 \]

In this simple transportation model with $m > 1$ distribution centers and $n > 1$ destinations, an activity is the making of a shipment from a specific distribution center to a specific destination. There would be $mn$ activities to be considered. We would therefore have $m$ restrictions relating to distribution centers and $n$ restrictions relating to destinations. But since distribution center capacity must equal to the total of requirements for all destinations and all shipments are non-negative, we would then have $m + n - 1$ effective restrictions and a minimum-cost set of routes will exist in which only $m + n - 1$ of the activities are used at positive levels.